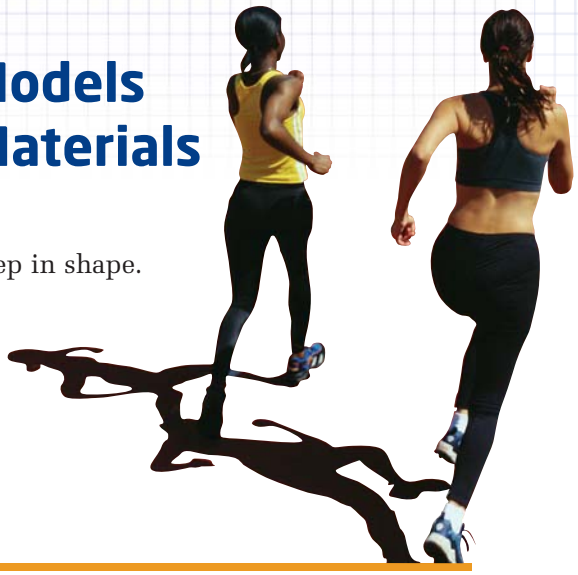
# 3.1 Build Algebraic Models Using Concrete Materials

Petra likes to run at the track to keep in shape. This year, to motivate herself, she will record her training progress visually. What are some ways that she can do this?



## Investigate

**Tools**
- algebra tiles
- linking cubes

### How can you model length, area, and volume using concrete materials?

#### Part A: Model Length

Petra's running record for the first 2 weeks of the year is shown below. Petra used algebra tiles to model each distance. She used the side length of a **unit tile** to represent 1 km.

**Algebra tiles** are tools that can be used to model measured quantities.

What other concrete materials could I use to represent one unit?

A unit tile is a square tile that measures 1 unit by 1 unit. It can be used as a counter.

unit tile

In this book, positive algebra tiles are green and negative algebra tiles are white.

On July 14, Petra ran 4 km.

| Date | Distance (km) | Progress |
|---|---|---|
| January 1 | 1 | |
| January 3 | 2 | |
| January 6 | 2 | |
| January 8 | 3 | |
| January 10 | 3 | |
| January 14 | 4 | |

A tile model is a good way of tracking Petra's progress visually.

One weekend, Petra cross-country skiied around a lake three times. She did not know the distance around the lake, so she used a **variable**, $x$, to represent it.

**variable**

- a quantity whose value can change (or vary)
- usually represented by a letter

An ***x*-tile** is a rectangular tile that is used to represent the variable $x$. It has the same width as the side length of a unit tile. You can use an $x$-tile to describe any unknown value.

The total distance can be modelled using three $x$-tiles:

$x$-tile

This means that on the weekend, Petra skiied a total of $3x$ kilometres, where $x$ is the distance around the lake.

**1.** Use tiles to represent each length.

**a)** 6 km

**b)** an unknown distance

**c)** an unknown distance and back

**d)** 3 km plus an unknown distance

**2.** Use tiles to model each **algebraic expression**.

**a)** 7 **b)** $4x$ **c)** $x + 3$ **d)** $3x + 2$

**algebraic expression**

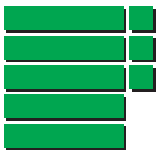
- a mathematical phrase made up of numbers and variables, connected by addition or subtraction operators
- can be used to model real-life situations
- $2x + 3$ is an algebraic expression

**3.** Write an algebraic expression for each model.

**a)** 

**b)** 

**c)** 

**d)** 

**4.** Create an algebraic expression of your own and build a tile model to represent it. Record the expression and the model.

**Literacy Connections**

Expressions are sometimes confused with equations. For example, $2x + 3$ is an expression, but $2x + 3 = 1$ is an equation. Equations contain an equal sign, expressions do not.

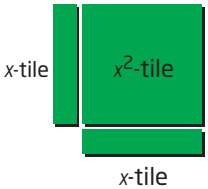
### Part B: Model Area

You can use an **$x^2$-tile** to represent an unknown area, measured in square units, if you let the side length of each tile represent $x$ units. Note that the side length of an $x^2$-tile is equal to the length of an $x$-tile.

1 4 9

(square units)

$x^2$-tile



This model shows that Petra mowed one square lawn 6 times over a summer. The total area mowed, in square metres, is $6x^2$, where $x$ is the side length, in metres, of the square lawn.

**5.** Use algebra tiles to represent each area.

**a)** 16 square units

**b)** 25 square units

**c)** $3x^2$

**d)** $5x^2$

**6.** Use algebra tiles to represent each expression.

**a)** $x^2 + 2$

**b)** $2x^2 + x$

**c)** $x^2 + 3x + 2$

**d)** $3x^2 + 5x + 1$

### Part C: Model Volume

**7.** A cube is a rectangular prism with length, width, and height all equal.

**a)** Use linking cubes to build a model of a cube that has a side length of 3 cm. Sketch your model.

**b)** What is the volume, in cubic centimetres, of this cube?

**c)** Express the volume of the cube as a power.

**8.** Repeat step 7 for a cube with a side length of 5 cm.

**9.** Describe other concrete materials you could use to build an algebraic model of volume.

**10.** Suppose you do not know the side length of a cube. You can use the variable $x$ to represent the side length.

**a)** Sketch the cube and label its length, width, and height.

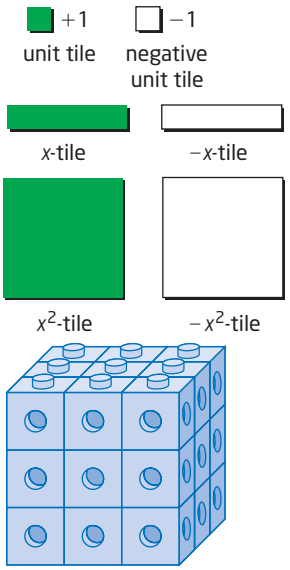
**b)** Write an algebraic expression for the area of one face of the cube.

**c)** Write an algebraic expression for the volume of the cube.

**11.** **Reflect** Describe how concrete materials can be used to build algebraic models of length, area, and volume. Use words, expressions, and diagrams to support your explanation.

## Key Concepts

- Concrete materials, such as algebra tiles and linking cubes, can be used to build algebraic models.
- You can build length models with algebra tiles.
- You can build area models with algebra tiles.
- You can build volume models with linking cubes.




## Communicate Your Understanding

**C1** State the length and width of each.

**a)** a unit tile

**b)** an $x$-tile

**c)** an $x^2$-tile

**C2** **a)** How are the length and width of a unit tile and an $x$-tile related?

**b)** How are the length and width of an $x$-tile and an $x^2$-tile related?

**C3** **a)** Suggest two other objects that could be used to model length.

**b)** Suggest two other objects that could be used to model area.

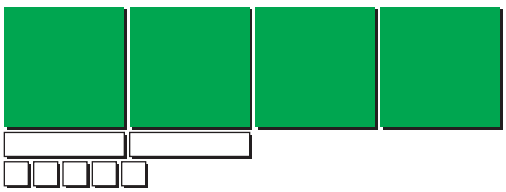
**c)** For your answers to parts a) and b), identify any advantages or disadvantages of each object.

**C4** Explain how concrete materials can be used to model each type of measurement. Include a diagram to support each explanation.

**a)** length **b)** area **c)** volume

## Practise

**1.** Which expression is represented by the algebra tile model?



**A** $4x^2 + 2x - 5$ **B** $-4x^2 - 2x - 5$

**C** $4x^2 - 2x - 5$ **D** $4x^2 + 2x + 5$

**Did You Know?**

The Greek mathematician Diophantus (about 250-275) was the first person to use a letter to represent an unknown.

**2.** Use tiles to model each algebraic expression.

**a)** $x^2 + 3x$

**b)** $2x^2 + 5$

**c)** $3x^2 + x + 2$

**d)** $x^2 + 2x + 4$

**3.** Write the algebraic expression represented by each model.

**a)**

**b)**

**c)**

**d)**

## Connect and Apply

**4.** Each unit tile represents 1 km that Miko rode her bicycle. Find each distance.

**a)**

**b)**

**c)**

**d)**

**5.** Create an algebraic expression of your own, using $x^2$-tiles, $x$-tiles, and unit tiles, and build a tile model to represent it. Record the expression and the model.

**6. a)** Build a volume model to represent a cube with length, width, and height all equal to 4 cm. Sketch the model and label the length, width, and height.

**b)** What is the volume? Write this as a power.

**c)** Write an expression for the area of one face as a power. Evaluate the area of one face.

**7.** A cube has a volume of 216 $cm^3$.

**a)** What side length of the cube would give this volume?

**b)** Determine the area of one face of the cube.

**8.** The area of one face of a cube is 49 $m^2$.

**a)** What side length of the cube would give this area?

**b)** Determine the volume of the cube.

## Extend

**9.** Build an area model using tiles that have length and width as indicated.

**a)** length $= x + 3$, width $= x$

**b)** length $= x + 4$, width $= x + 1$

**10.** A cube has a volume of 8 $cm^3$. Find the total surface area of all six faces.

**11. Math Contest** Mersenne numbers are numbers of the form $2^n - 1$. Father Marin Mersenne (1588–1648) was especially interested in prime numbers of this form. One conjecture about Mersenne numbers is that numbers of the form $2^p - 1$ are prime if $p$ is prime. Investigate this conjecture and write a brief report of your findings.

**12. Math Contest** Find the smallest possible value of $a^b + c^d + e^f$ if $a$, $b$, $c$, $d$, $e$, and $f$ are all different and are chosen from the values 1, 2, 3, 4, 5, and 6.

**13. Math Contest** When $30^{40} \times 40^{30}$ is written in expanded form, the number of zeros at the end of the number is

**A** 30

**B** 40

**C** 70

**D** 120

**E** 1200

**14. Math Contest** Fermat numbers are numbers of the form $2^{2^n} + 1$. Pierre de Fermat (1601–1665) conjectured that all numbers of this form are primes. Investigate this conjecture. Write a brief report of your findings.